

## Pregled formula matematičke statistike i statističke tablice

### Statistike

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Uzoračka srednja vrednost  $\bar{X}_n = \frac{1}{n} \sum_{k=1}^n X_k$

Uzoračka disperzija  $\bar{S}_n^2 = \frac{1}{n} \sum_{k=1}^n X_k^2 - (\bar{X}_n)^2$

Popravljen uzoračka disperzija  $\hat{S}_n^2 = \frac{n}{n-1} \bar{S}_n^2$

Uzoračka standardna devijacija  $\bar{S}_n = \sqrt{\bar{S}_n^2}$

Uzorački koeficijent korelacije obeležja  $X$  i  $Y$   $R_{XY} = \frac{\frac{1}{n} \sum_{k=1}^n X_k Y_k - \bar{X}_n \bar{Y}_n}{\sqrt{\bar{S}_X^2 \bar{S}_Y^2}}$

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### Tačkaste ocene parametara nekih raspodela

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$S_n : \mathcal{B}(n, p) \quad \hat{p} = \frac{S_n}{n}$

$X : \mathcal{P}(\lambda) \quad \hat{\lambda} = \bar{X}_n$

$X : \mathcal{N}(m, \sigma^2) \quad \hat{m} = \bar{X}_n, \hat{\sigma}^2 = \bar{S}_n^2$

$X : \mathcal{E}(\lambda) \quad \hat{\lambda} = \frac{1}{\bar{X}_n}$

$X : \mathcal{U}(a, b) \quad \hat{a} = X_{\min} - \frac{X_{\max} - X_{\min}}{n-1}, \hat{b} = X_{\max} + \frac{X_{\max} - X_{\min}}{n-1}$

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## Intervali poverenja

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$$\mathbf{X} : \mathcal{N}(\mathbf{m}, \sigma^2)$$

Interval poverenja za parameter  $m$ , ako je  $\sigma^2$  poznato

$$I_m = \left[ \bar{X}_n - z_\beta \frac{\sigma}{\sqrt{n}}, \bar{X}_n + z_\beta \frac{\sigma}{\sqrt{n}} \right] \quad \Phi(z_\beta) = \frac{\beta}{2}$$

Interval poverenja za parameter  $m$ , ako je  $\sigma^2$  nepoznato

$$I_m = \left[ \bar{X}_n - t_{n-1, 1-\beta} \frac{\bar{S}_n}{\sqrt{n-1}}, \bar{X}_n + t_{n-1, 1-\beta} \frac{\bar{S}_n}{\sqrt{n-1}} \right]$$

Jednostrani interval poverenja za parameter  $\sigma^2$

$$I_{\sigma^2} = \left[ 0, \frac{n\bar{S}_n^2}{\chi_{n-1, \beta}^2} \right]$$

Dvostrani interval poverenja za parameter  $\sigma^2$

$$I_{\sigma^2} = \left[ \frac{n\bar{S}_n^2}{\chi_{n-1, \frac{1-\beta}{2}}^2}, \frac{n\bar{S}_n^2}{\chi_{n-1, \frac{1+\beta}{2}}^2} \right]$$


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## Testiranje statističkih hipoteza

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$$\mathbf{S}_n : \mathcal{B}(\mathbf{n}, \mathbf{p})$$

$$H_0(p=p_0) \quad H_1(p \neq p_0) \quad C = (-\infty, -z_\alpha) \cup (z_\alpha, +\infty), \quad \Phi(z_\alpha) = \frac{1-\alpha}{2}$$

$$H_0(p=p_0) \quad H_1(p > p_0) \quad C = (z_\alpha, +\infty), \quad \Phi(z_\alpha) = \frac{1}{2} - \alpha$$

$$H_0(p=p_0) \quad H_1(p < p_0) \quad C = (-\infty, -z_\alpha), \quad \Phi(z_\alpha) = \frac{1}{2} - \alpha$$

$$Y_n = \frac{S_n - np_0}{\sqrt{np_0(1-p_0)}}$$

$$Y_n \in C \Rightarrow H_0 \text{ odbacujemo}, \quad Y_n \notin C \Rightarrow H_0 \text{ prihvatamo}$$


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$$\mathbf{X} : \mathcal{N}(\mathbf{m}, \sigma^2)$$

$\sigma^2$  poznato

$$H_0(m=m_0) \quad H_1(m \neq m_0) \quad C = (-\infty, -z_\alpha) \cup (z_\alpha, +\infty), \quad \Phi(z_\alpha) = \frac{1-\alpha}{2}$$

$$H_0(m=m_0) \quad H_1(m > m_0) \quad C = (z_\alpha, +\infty), \quad \Phi(z_\alpha) = \frac{1}{2} - \alpha$$

$$H_0(m=m_0) \quad H_1(m < m_0) \quad C = (-\infty, -z_\alpha), \quad \Phi(z_\alpha) = \frac{1}{2} - \alpha$$

$$Z_n = \frac{\bar{X}_n - m_0}{\sigma} \sqrt{n}$$

$$Z_n \in C \Rightarrow H_0 \text{ odbacujemo}, \quad Z_n \notin C \Rightarrow H_0 \text{ prihvatamo}$$


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**X** :  $\mathcal{N}(\mathbf{m}, \sigma^2)$

$\sigma^2$  nepoznato, a  $n > 30$

$H_0(m = m_0)$	$H_1(m \neq m_0)$	$C = (-\infty, -z_\alpha) \cup (z_\alpha, +\infty), \Phi(z_\alpha) = \frac{1-\alpha}{2}$
$H_0(m = m_0)$	$H_1(m > m_0)$	$C = (z_\alpha, +\infty), \Phi(z_\alpha) = \frac{1}{2} - \alpha$
$H_0(m = m_0)$	$H_1(m < m_0)$	$C = (-\infty, -z_\alpha), \Phi(z_\alpha) = \frac{1}{2} - \alpha$

$$Z_n = \frac{\bar{X}_n - m_0}{\bar{S}_n} \sqrt{n-1}$$

$Z_n \in C \Rightarrow H_0$  odbacujemo,  $Z_n \notin C \Rightarrow H_0$  prihvatamo

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**X** :  $\mathcal{N}(\mathbf{m}, \sigma^2)$

$\sigma^2$  nepoznato, a  $n \leq 30$

$H_0(m = m_0)$	$H_1(m \neq m_0)$	$C = (-\infty, -t_{n-1, \alpha}) \cup (t_{n-1, \alpha}, +\infty)$
$H_0(m = m_0)$	$H_1(m > m_0)$	$C = (t_{n-1, 2\alpha}, +\infty)$
$H_0(m = m_0)$	$H_1(m < m_0)$	$C = (-\infty, -t_{n-1, \alpha})$

$$t_{n-1} = \frac{\bar{X}_n - m_0}{\bar{S}_n} \sqrt{n-1}$$

$t_{n-1} \in C \Rightarrow H_0$  odbacujemo,  $t_{n-1} \notin C \Rightarrow H_0$  prihvatamo

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**X** :  $\mathcal{N}(\mathbf{m}, \sigma^2)$

$H_0(\sigma^2 = \sigma_0^2)$	$H_1(\sigma^2 \neq \sigma_0^2)$	$C = (0, \chi_{n-1, 1-\frac{\alpha}{2}}^2) \cup (\chi_{n-1, \frac{\alpha}{2}}^2, +\infty)$
$H_0(\sigma^2 = \sigma_0^2)$	$H_1(\sigma^2 > \sigma_0^2)$	$C = (\chi_{n-1, \alpha}^2, +\infty)$
$H_0(\sigma^2 = \sigma_0^2)$	$H_1(\sigma^2 < \sigma_0^2)$	$C = (0, \chi_{n-1, 1-\alpha}^2)$

$$\chi_{n-1}^2 = \frac{n\bar{S}_n^2}{\sigma_0^2}$$

$\chi_{n-1}^2 \in C \Rightarrow H_0$  odbacujemo,  $\chi_{n-1}^2 \notin C \Rightarrow H_0$  prihvatamo

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$$\mathbf{X} : \mathcal{N}(\mathbf{m}_1, \sigma^2) \quad \mathbf{Y} : \mathcal{N}(\mathbf{m}_2, \sigma^2)$$

$$\begin{array}{lll} H_0(m_1 = m_2) & H_1(m_1 \neq m_2) & C = (-\infty, -t_{n+m-2, \alpha}) \cup (t_{n+m-2, \alpha}, +\infty) \\ H_0(m_1 = m_2) & H_1(m_1 > m_2) & C = (t_{n+m-2, 2\alpha}, +\infty) \\ H_0(m_1 = m_2) & H_1(m_1 < m_2) & C = (-\infty, -t_{n+m-2, \alpha}) \end{array}$$

$$t_{n+m-2} = \frac{\bar{X}_n - \bar{Y}_m}{\sqrt{n\bar{S}_n^2 + m\bar{S}_m^2}} \sqrt{\frac{nm}{n+m}} (n+m-2)$$

$$t_{n+m-2} \in C \Rightarrow H_0 \text{ odbacujemo}, \quad t_{n+m-2} \notin C \Rightarrow H_0 \text{ prihvatamo}$$


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$$(\mathbf{X}, \mathbf{Y}) : \mathcal{N}(\mathbf{m}_1, \mathbf{m}_2, \sigma_1^2, \sigma_2^2, \rho)$$

$$\begin{array}{lll} H_0(\rho = 0) & H_1(\rho \neq 0) & C = (-\infty, -t_{n-2, \alpha}) \cup (t_{n-2, \alpha}, +\infty) \\ H_0(\rho = 0) & H_1(\rho > 0) & C = (t_{n-2, 2\alpha}, +\infty) \\ H_0(\rho = 0) & H_1(\rho < 0) & C = (-\infty, -t_{n-2, \alpha}) \end{array}$$

$$t_{n-2} = \frac{R_{XY}}{\sqrt{1 - R_{XY}^2}} \sqrt{n-2}$$

$$t_{n-2} \in C \Rightarrow H_0 \text{ odbacujemo}, \quad t_{n-2} \notin C \Rightarrow H_0 \text{ prihvatamo}$$


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**Prava linearne regresije**  $y = \hat{a}x + \hat{b}$

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$$\hat{a} = \frac{\frac{1}{n} \sum_{k=1}^n X_k Y_k - \bar{X}_n \bar{Y}_n}{\frac{1}{n} \sum_{k=1}^n X_k^2 - (\bar{X}_n)^2}, \quad \hat{b} = \bar{Y}_n - \hat{a} \bar{X}_n$$


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Tablica II: Studentova  $t$  raspodela

n	$\alpha$							
	0.80	0.60	0.40	0.20	0.10	0.05	0.02	0.01
1	0.325	0.727	1.376	3.078	6.314	12.706	31.823	63.657
2	0.289	0.617	1.061	1.886	2.920	4.403	6.965	9.925
3	0.277	0.584	0.978	1.638	2.353	3.182	4.531	5.841
4	0.271	0.569	0.941	1.533	2.132	2.776	3.747	4.604
5	0.267	0.559	0.920	1.476	2.015	2.571	3.365	4.032
6	0.265	0.553	0.906	1.440	1.943	2.447	3.143	3.707
7	0.263	0.549	0.896	1.415	1.895	2.365	2.998	3.499
8	0.262	0.546	0.889	1.397	1.860	2.306	2.896	3.355
9	0.261	0.543	0.883	1.383	1.833	2.262	2.821	3.250
10	0.260	0.542	0.879	1.372	1.812	2.228	2.764	3.169
11	0.260	0.540	0.876	1.363	1.796	2.201	2.718	3.106
12	0.259	0.539	0.873	1.356	1.782	2.179	2.681	3.055
13	0.259	0.538	0.870	1.350	1.771	2.160	2.650	3.012
14	0.258	0.537	0.868	1.345	1.761	2.145	2.624	2.977
15	0.258	0.536	0.866	1.341	1.753	2.131	2.602	2.947
16	0.258	0.535	0.865	1.337	1.746	2.120	2.583	2.921
17	0.257	0.534	0.863	1.333	1.740	2.110	2.567	2.898
18	0.257	0.534	0.862	1.330	1.734	2.101	2.552	2.878
19	0.257	0.533	0.861	1.328	1.729	2.093	2.539	2.861
20	0.257	0.533	0.860	1.325	1.725	2.086	2.528	2.845
21	0.257	0.532	0.859	1.323	1.721	2.080	2.518	2.831
22	0.256	0.532	0.858	1.321	1.717	2.074	2.508	2.819
23	0.256	0.532	0.858	1.319	1.714	2.069	2.500	2.807
24	0.256	0.531	0.857	1.318	1.711	2.064	2.492	2.797
25	0.256	0.531	0.856	1.316	1.708	2.060	2.485	2.787
26	0.256	0.531	0.856	1.315	1.706	2.056	2.479	2.779
27	0.256	0.531	0.855	1.314	1.703	2.052	2.473	2.771
28	0.256	0.530	0.855	1.313	1.701	2.048	2.467	2.763
29	0.256	0.530	0.854	1.311	1.699	2.045	2.462	2.756
30	0.256	0.530	0.854	1.310	1.697	2.042	2.457	2.750
40	0.255	0.529	0.851	1.303	1.684	2.021	2.423	2.704
60	0.254	0.527	0.848	1.296	1.671	2.000	2.390	2.660
120	0.254	0.526	0.845	1.289	1.658	1.980	2.358	2.617
$\infty$	0.253	0.524	0.842	1.282	1.645	1.960	2.326	2.576

Tablica III:  $\chi^2$  raspodela

n	$\alpha$									
	0.90	0.80	0.70	0.50	0.30	0.20	0.10	0.05	0.02	0.01
1	0.016	0.064	0.148	0.455	1.074	1.642	2.706	3.841	5.412	6.635
2	0.211	0.446	0.713	1.386	2.408	3.219	4.605	5.991	7.824	9.210
3	0.584	1.005	1.424	2.366	3.665	4.642	6.251	7.815	9.837	11.345
4	1.064	1.649	2.195	3.357	4.878	5.989	7.779	9.488	11.668	12.277
5	1.610	2.343	3.000	4.351	6.064	7.289	9.236	11.070	13.388	15.086
6	2.204	3.070	3.828	5.348	7.231	8.558	10.645	12.592	15.033	16.812
7	2.833	3.822	4.671	6.346	8.383	9.803	12.017	14.067	16.622	18.475
8	3.490	4.594	5.557	7.344	9.524	11.030	13.362	15.507	18.168	20.090
9	4.168	5.380	6.393	8.343	10.656	12.242	14.684	16.919	19.679	21.666
10	4.865	6.179	7.267	9.342	11.781	13.442	15.987	18.307	21.161	23.209
11	5.579	6.989	8.148	10.341	12.899	14.631	17.276	19.675	22.618	24.725
12	6.304	7.807	9.034	11.340	14.011	15.812	18.549	21.026	24.054	26.217
13	7.042	8.634	9.926	12.340	15.119	16.985	19.812	22.362	25.472	27.688
14	7.790	9.467	10.821	13.339	16.222	18.151	21.064	23.685	26.873	29.141
15	8.547	10.307	11.721	14.339	17.322	19.311	22.307	24.996	28.259	30.578
16	9.312	11.152	12.624	15.338	18.418	20.465	23.542	26.296	29.633	32.000
17	10.085	12.002	13.531	16.338	19.511	21.615	24.769	27.587	30.995	33.409
18	10.865	12.857	14.440	17.338	20.601	22.860	25.989	29.869	32.346	34.805
19	11.651	13.716	15.352	18.338	21.689	23.900	27.204	30.144	33.687	36.191
20	12.443	14.578	16.266	19.338	22.775	25.038	28.412	31.410	35.020	37.564
21	13.240	15.445	17.182	20.337	23.958	26.171	29.615	32.671	36.343	38.932
22	14.041	16.314	18.101	21.337	24.839	27.301	30.813	33.924	37.659	40.289
23	14.848	17.187	19.021	22.337	26.018	28.429	32.007	35.172	38.968	41.638
24	15.659	18.062	19.943	23.337	27.096	29.553	33.196	36.415	40.270	42.980
25	16.473	18.940	20.867	24.337	28.172	30.675	34.382	37.652	41.564	44.314
26	17.292	19.820	21.792	25.336	29.246	31.795	35.563	38.885	42.852	45.642
27	18.114	20.703	22.719	26.336	30.319	32.912	36.741	40.113	44.143	46.963
28	18.739	21.588	23.647	27.336	31.391	34.027	37.916	41.337	45.418	48.278
29	18.768	22.475	24.577	28.336	32.461	35.139	39.087	42.557	46.698	49.588
30	20.599	23.364	25.508	29.336	33.530	36.250	40.256	43.773	47.962	50.892