1 Circular Pascal Arrays and Corridor Paths

1.1 Circular Pascal Arrays

We begin by defining the *circular Pascal arrays* (one for each integer $d \geq 2$) and explore some of their amazing properties. By *Pascal array*, we mean something a little more general than the familiar Pascal's triangle. In what follows, we interpret the binomial coefficient $\binom{n}{k}$ as the coefficient of x^n in the expansion,

$$(1+x)^n = \sum_{k \in \mathbb{Z}} \binom{n}{k} x^k,$$

where we understand $\binom{n}{k} = 0$ if $k < 0$ or $k > n$. We also use the convention that $\mathbb{N}$ denotes the set of non-negative integers, $\{0, 1, 2, 3, \ldots\}$.

$$\sigma_{n,k} = \sigma_{n,k}^{(d)} = \sum_{j \in \mathbb{Z}} \binom{n}{k+dj}$$

$2^{\lfloor n/2 \rfloor}$,$2^{\lceil n/2 \rceil}$

$$\mathbf{e}_0 = (\ldots, 0, 1, \underbrace{0, \ldots, 0}_{d-1 \text{ zeros}}, \mathbf{1}, \underbrace{0, \ldots, 0}_{d-1 \text{ zeros}}, 1, 0, \ldots)$$

By the very definition of the $\mathbf{v}_n$ we have

$$v_{n,0} = v_{n,\pm d} = v_{n,\pm 2d} = \ldots = 0.$$

$$H(X) = -\sum_{y \in Y} \sum_{\substack{x \\ g(x)=y}} p(x) \log p(x).$$

Moreover, $v_{n,k}$ are antisymmetric about $k = 0$, i.e.,

$$v_{n,-k} = -v_{n,k} \text{ for all } k \in \mathbb{Z}. \tag{1}$$

By (1) and the fact that $(L+R)^n = [L(I+R^2)]^n = L^n(I+R^2)^n$, we get

$$q_{n,n+y_0+j} = \begin{cases} v_{n,0} = 0, & \text{if } j \equiv 0 \pmod{2d}, \\ v_{n,j} \geq 0, & \text{if } j \equiv 1, 2, \ldots, d-1 \pmod{2d}, \\ v_{n,d} = 0, & \text{if } j \equiv d \pmod{2d}, \\ v_{n,j} \leq 0, & \text{if } j \equiv d+1, d+2, \ldots, 2d-1 \pmod{2d}. \end{cases} \tag{2}$$

Then by (2) we have

$$\begin{aligned} D(a,b;s,t) =& \sigma_{a+b,b-s} - \sigma_{a+b,b-s+1} \\ =& \sum_{k \in \mathbb{Z}} \left[\binom{a+b}{b-s+dk} + \binom{a+b}{b-s-1+dk} + \cdots + \binom{a+b}{b-s-(-s)+dk} \right] \\ &- \sum_{k \in \mathbb{Z}} \left[\binom{a+b}{b-s+1+dk} + \binom{a+b}{b-s+dk} + \cdots + \binom{a+b}{b-s+1-(-s)+dk} \right] \\ =& \sum_{k \in \mathbb{Z}} \left[\binom{a+b}{a-k(t-s+2)} - \binom{a+b}{a-k(t-s+2)+s-1} \right]. \end{aligned} \tag{3}$$

$$
\begin{aligned}
c_{n,y_0}^{(\infty)} = c_{n,y_0}^{(n+y+0)} &= p_{n,n+y_0}^{(n+y_0+2)} - p_{n,n+y_0+(n+y_0+2)}^{(n+y_0+2)} &(4)\\
&= p_{n,n+y_0}^{(n+y_0+2)} - p_{n,2n+2y_0+2}^{(n+y_0+2)} &(5)\\
&= \tilde{\sigma}_{n,\lfloor (n+y_0)/2\rfloor}^{(n+y_0+2)} - \hat{\sigma}_{n,n+y_0+1}^{(n+y_0+2)} &(6)
\end{aligned}
$$

2 Uredjeni skupovi i mreže

Za mrežu L i $a \in L$, podmreže

$$[a) = \{x \in L | a \leq x\} \qquad \text{i} \qquad (a] = \{x \in L | x \leq a\}$$

su *poluotvoreni intervali* mreže L.

Neka je $(L, \wedge, \vee, 0, 1)$ kompletna mreža. Tada je *kvantal* $(L, \wedge, \vee, *, 0, 1)$ algebra u kojoj binarna operacija $* : L^2 \to L$ zadovoljava uslove

$$
\begin{aligned}
u * (v * w) &= (u * v) * w,\\
u * (\textstyle\bigvee_{i\in I} v_i) &= \displaystyle\bigvee_{i\in I} (u * v_i),\\
(\textstyle\bigvee_{i\in I} u_i) * v &= \displaystyle\bigvee_{i\in I} (u_i * v),
\end{aligned}
$$

za svako $u, v, w \in L$ i $u_i, v_i \in L$ za svako $i \in I$. U svakoj kompletno reziduiranoj mreži sledeća tvrdjenja su tačna, za svaki indeksni skup I:

$$
\begin{aligned}
a \otimes \bigvee_{i\in I} b_i &= \bigvee_{i\in I} (a \otimes b_i), &(7)\\
a \to \bigwedge_{i\in I} b_i &= \bigwedge_{i\in I} (a \to b_i),\\
\bigvee_{i\in I} a_i \to b &= \bigwedge_{i\in I} (a_i \to b),\\
a \otimes \bigwedge_{i\in I} b_i &\leq \bigwedge_{i\in I} (a \otimes b_i), &(8)
\end{aligned}
$$

Neka je $\mathcal{L}$ Gödel-ova struktura, i skupovi U i V takvi da je $|U| = 3$ i $|V| = 2$. Neka su $A_1, A_2 \in L^{U\times U}$, $B_1, B_2 \in L^{V\times V}$, $N \in L^{U\times V}$ fazi relacije zadate preko sledećih fazi matrica:

$$
N = \begin{bmatrix} 1 & 1\\ 1 & 1\\ 1 & 1 \end{bmatrix}, \quad
A_1 = \begin{bmatrix} 1 & 0.3 & 0.4\\ 0.5 & 1 & 0.3\\ 0.4 & 0.6 & 1 \end{bmatrix}, \quad
A_2 = \begin{bmatrix} 0.5 & 0.6 & 0.2\\ 0.6 & 0.3 & 0.4\\ 0.7 & 0.7 & 1 \end{bmatrix},
$$

$$
B_1 = \begin{bmatrix} 1 & 0.6\\ 0.6 & 0.7 \end{bmatrix}, \quad
B_2 = \begin{bmatrix} 0.6 & 0.6\\ 0.7 & 1 \end{bmatrix}.
$$

Matrica:

$$A = \begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{n1} & a_{n2} & \cdots & a_{nn} \end{pmatrix}$$

The generalized Pascal matrix $\mathcal{P}_n[r; x]$ of order 5 is equal to

$$\mathcal{P}_5[r; x] = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ \binom{r}{1}x & 1 & 0 & 0 & 0 \\ \binom{r+1}{2}x^2 & \binom{r+1}{1}x & 1 & 0 & 0 \\ \binom{r+2}{3}x^3 & \binom{r+2}{2}x^2 & \binom{r+2}{1}x & 1 & 0 \\ \binom{r+3}{4}x^4 & \binom{r+3}{3}x^3 & \binom{r+3}{2}x^2 & \binom{r+3}{1}x & 1 \end{bmatrix}$$