1 Circular Pascal Arrays and Corridor Paths

Definition 1. *The **Pascal array** is the array whose row n, column k entry is equal to $\binom{n}{k}$, where $n \in \mathbb{N}$ and $k \in \mathbb{Z}$.*

Definition 2. *Fix an integer $d \geq 2$. The **circular Pascal array of order** d is the array whose row n, column k entry, $\sigma_{n,k}^{(d)}$ (or just $\sigma_{n,k}$ when the context is clear), is the (finite) sum:*

$$\sigma_{n,k} = \sigma_{n,k}^{(d)} = \sum_{j \in \mathbb{Z}} \binom{n}{k + dj}, \tag{1}$$

where $n \in \mathbb{N}$ and $k \in \mathbb{Z}$.

Lemma 1.1. *For all $n \geq 0$, $q_n = (I + R^2)^n q_0$.*

Proof. Because $I + R^2$ and $D = I - L$ commute, a simple induction gives the result. $\square$

Theorem 1.2. *For fixed $d \geq 2$, and $0 \leq y_0 \leq d - 2$, the n^{th} corridor number of order $d - 2$ is given by $c_{n,y_0} = p_{n,n+y_0} - p_{n,n+y_0+d}$.*

Proof. Let $p_{n,k}$, *resp.* $q_{n,k}$, be the k^{th} entry of p_n, resp. q_n. By definition, $c_{n,y0} = \sum_{k=0}^{d-1} v_{n,k}$. So by Lemma 1.1, since $q_{n,n+y_0+j} = p_{n,n+y_0+j} - p_{n,n+y_0+j+1}$ is essentially a negative discrete derivative, it follows that $p_{n,n+y_0}$ is a maximum value of p_n, and $p_{n,n+y_0+d}$ is a minimum value of p_n, which completes the proof. $\square$

Corollary 1.3. *The $(d-2)$-corridor number c_n (i.e., when $y_0 = 0$) equals the range of the n^{th} row of the (standard) circular Pascal array or order d.*

2 Citing some references

Figure 1 shows a koala.

When the allowable moves in a corridor include remaining at the same level, the paths are often called "Motzkin." (see, for example [3]) The structure we have already set up extends easily to such Motzkin paths, and the related structures are called Göder structures.

Upon the up-sampling, the extreme values fell nicely on diagonals of the circular Pascal arrays. Finally, with the introduction of the dual corridor, we found the strong connection between the two structures. Using the most basic properties of a few simple operators, we arrived at our main result, which easily led to a few nontrivial lattice path results - please refer to [1–3, 5] for more details.

Figure 1: A koala.

References

[1] C. Kicey and K. Klimko, Some geometry of Pascal's triangle, *Pi Mu Epsilon Journal*, 13(4):229–245, 2011.

[2] Leslie Lamport, *LaTeX: a document preparation system*. Addison Wesley, Massachusetts, 2nd edition, 1994.

[3] Lajos Takács, Ballot problems, *Zeitschrift für Wahrscheinlichkeitstheorie und Verwandte Gebiete*, 1(2):154–158, 1962.

[4] Eric W. Weisstein. Trinomial triangle. From MathWorldA Wolfram Web Resource. `http://mathworld.wolfram.com/TrinomialTriangle.html`

[5] Eric W. Weisstein. Motzkin triangle. From MathWorldA Wolfram Web Resource.