PDJ drugog reda – II deo

Rimanova metoda

Rimanovom metodom rešiti sledeće Košijeve probleme:

1. $u_{xy} + \dfrac{u_x + u_y}{x+y} = 2, \quad 0 < x, y < \infty$

$u\big|_{y=x} = x^2, \quad u_x\big|_{y=x} = x + 1$

2. $u_{xy} + 2u_x + u_y + 2y = 1, \quad 0 < x, y < 1$

$u\big|_{x+y=1} = x, \quad u_x\big|_{x+y=1} = x$

3. $x^2 u_{xx} - y^2 u_{yy} - 2y u_y = 0, \quad x > 1,\ y > 0$

$u\big|_{x=1} = y, \quad u_x\big|_{x=1} = y > 0$

4. $u_{xx} + 4u_{xy} - 5u_{yy} + u_x - u_y = 0, \quad x > 0,\ y > 0$

$u\big|_{y=0} = f(x), \quad u_y\big|_{y=0} = g(x)$

5. Pokazati da jednačina

$$u_{xy} - \frac{2}{(x+y)^2} u = 0$$

ima Rimanovu funkciju oblika

$$v(x, y; x_0, y_0) = \frac{(x + y_0)(x_0 + y)}{(x+y)(x_0+y_0)} + \frac{(x - x_0)(y - y_0)}{(x+y)(x_0+y_0)}.$$

Rešiti datu jednačinu za početne vrednosti

$$u|_{y=x} = 0, \quad u_x|_{y=x} = x^2.$$