

Furijeov metod za PDJ hiperboličnog i parabolichnog tipa

Rešiti sledeće Šturm-Liuvilove granične probleme::

- (I) $X''(x) + \lambda X(x) = 0, \quad x \in (0, l), \quad X(0) = X(l) = 0;$
- (II) $X''(x) + \lambda X(x) = 0, \quad x \in (0, l), \quad X(0) = X'(l) = 0;$
- (III) $X''(x) + \lambda X(x) = 0, \quad x \in (0, l), \quad X'(0) = X(l) = 0;$
- (IV) $X''(x) + \lambda X(x) = 0, \quad x \in (0, l), \quad X'(0) = X'(l) = 0;$
- (V) $X''(x) + \lambda X(x) = 0, \quad x \in (0, l), \quad X'(0) = X'(l) + h X(l) = 0, \quad h > 0;$
- (VI) $X''(x) + \lambda X(x) = 0, \quad x \in (0, l), \quad X'(0) - h X(0) = X'(l) = 0, \quad h > 0;$
- (VII) $X''(x) + \lambda X(x) = 0, \quad x \in (0, l), \quad X'(0) - h X(0) = X'(l) + h X(l) = 0, \quad h > 0.$

Rešiti sledeće mešovite probleme PDJ hiperboličnog tipa:

1. $u_{tt} = a^2 u_{xx}, \quad 0 < x < l, \quad t > 0$
 $u_x(0, t) = u_x(l, t) = 0,$
 $u(x, 0) = x, \quad u_t(x, 0) = 1.$

2. $u_{tt} - 3u_t = u_{xx} + 2u_x, \quad 0 < x < \pi, \quad t > 0$
 $u(0, t) = u(\pi, t) = 0,$
 $u(x, 0) = e^{-x} \sin x, \quad u_t(x, 0) = 0.$

3. $u_{tt} - 2u_t = u_{xx} + 4t \sin x, \quad 0 < x < \frac{\pi}{2}, \quad t > 0$
 $u(0, t) = u_x\left(\frac{\pi}{2}, t\right) = 0,$
 $u(x, 0) = 0, \quad u_t(x, 0) = \sin x.$

4. $u_{tt} = u_{xx} + 6xt, \quad 0 < x < 1, \quad t > 0$
 $u(0, t) = t + 1, \quad u(1, t) = t^3 + 2,$
 $u(x, 0) = x + 1, \quad u_t(x, 0) = 1 - x^2.$

5. $u_{tt} = u_{xx} + 2\left(x + \frac{1}{h}\right), \quad 0 < x < l, \quad t > 0$
 $u_x(0, t) - h u(0, t) = t, \quad u_x(l, t) = t^2,$
 $u(x, 0) = f(x), \quad u_t(x, 0) = g(x).$

6. $u_{tt} = u_{xx} + t \cos 2\pi x - 2t, \quad 0 < x < 1, t > 0$
 $u_x(0, t) = t, \quad u_x(1, t) = 3t,$
 $u(x, 0) = \cos \pi x, \quad u_t(x, 0) = x^2 + x.$

7. $u_{tt} + u_t = u_{xx} + x, \quad 0 < x < 1, t > 0$
 $u_x(0, t) = u(1, t) = 0,$
 $u(x, 0) = \frac{1}{6}(1 - x^3), \quad u_t(x, 0) = 4 \cos \frac{\pi x}{2} \cos \pi x.$

Rešiti sledeće mešovite probleme PDJ paraboličnog tipa:

1. $u_t = 4u_{xx} - 2u, \quad 0 < x < \pi, t > 0$
 $u_x(0, t) = u_x(\pi, t) = 0,$
 $u(x, 0) = \cos \frac{3x}{2}.$

2. $u_t = 4u_{xx}, \quad 0 < x < l, t > 0$
 $u(0, t) = T, \quad u_x(l, t) + hu(l, t) = U, \quad h > 0$
 $u(x, 0) = 0.$

2. $u_t = u_{xx} + u - x + \sin 2x \cos x, \quad 0 < x < \pi/2, t > 0$
 $u(0, t) = 0, \quad u_x(\pi/2, t) = 1,$
 $u(x, 0) = x.$

4. $u_t = u_{xx} + u + xt(2 - t) + 2 \cos t \quad 0 < x < \pi, t > 0$
 $u_x(0, t) = u_x(\pi, t) = t^2,$
 $u(x, 0) = \cos 2x.$